Numerical modeling of physically nonlinear deformation elements of construction

Askhad M. Polatov, Nodira A. Nodirjanova

Abstract— In paper the problem definition, decision method, algorithm and the software of process of nonlinear deformation elements of constructional materials are given. On the basis of numerical modeling process of reduction of tension due to change of form contour of a design is researched.

Index Terms— numerical model, software, computer experiment, constructional materials, elastic-plastic state, toughness.

1 Introduction

One of the main directions of scientific and technical researches is the creation of adequate numerical models of functioning elements of structural. Analysis of the strain state of structural elements of materials with complex geometry was a prerequisite for the development of numerical modeling using computer technology. In this regard, the urgent problem is creation a mathematically-based numerical models, computational algorithms and specialized software to create tools for calculation of construction elements.

Using of numerical models in three-dimensional tasks is important as analytical methods allow us to find solutions of private tasks, mainly for elementary areas. Spatial elements of designs have the complex configuration including different or artificially introduced in structure of material cavities for simplification of weight. For determination of operability of such designs it is necessary to consider the existing geometrical features resulting in concentration of tension and formation of areas with plastic deformations.

It is known that by constructive changes it is possible to reach improvement in distribution of tension and to achieve increase in durability of a design [1]. We will note that the above described effect on the example of increase of fatigue durability for round cores with cross openings is given in paper [2]. Influence of unloading cracks on a tension of the pedigree massif in vicinities of development is described in paper [3]. Numerical modeling of an elastic-plastic condition of elements of designs allows analyzing influence of a site of various geometrical features on the intense deformed condition of designs by means of computing experiments.

At present, an effective method of numerical analysis of constructions is the finite element method (FEM). Efficiency of method depends on existence of the developed software which creation is the most labor-consuming part of realization of a method. The considering of matrix transformations of a method and widespread application of computer technologies promoted usage of FEM in the solution of applied tasks. FEM provides a solution to the three-dimensional elastic-plastic problems of deformation of elements of structural with complex geometry. In this regard development of numerical model and computing algorithms of the solution for three-dimensional elastic-plastic tasks is actual.

Additively the creation of specialized software systems which will automate process of research and projection of new constructions is significant.

The numerical model of deformations process of constructional elements of designs, algorithm of the decision and a complex of programs for the solution of problems of physically nonlinear deformation of three-dimensional isotropic bodies are given in paper [4-7] on the basis of FEM and the theory of small elastic-plastic deformations of Ilyushin A.A.

2 Problem definition

Statement of a regional task of the theory of elasticity for anisotropic bodies includes:

balance equations, $\sigma_{ij,j} + X_i = 0, \quad x_i \in V$, (1)

the generalized Hooke's law, $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$, (2)

Cauchy's ratios, $\varepsilon_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right)$ (3)

and regional conditions:

$$u_i\big|_{\Sigma_1} = u_i^o, \quad x_i \in \Sigma_1,$$

$$\sum_{j=1}^{3} \sigma_{ij} n_j\big|_{\Sigma_2} = S_i^o, \quad x_i \in \Sigma_2, \quad (4)$$

where - u_i components of a vector of movements;

X_i, S_i^0 - volume and superficial forces;

Σ_1, Σ_2 - the composed volume Σ surfaces V;

n_j - external normal to a volume Σ_2 surface V;

C_{ijkl} - tensor of elastic constants.

3 Decision method

The defining ratio of the deformation theory of plasticity of Ilyushin A.A. for isotropic bodies is represented in the form of the generalized Hooke's law (2) and registers in a look:

$$\sigma_{ij} = (\lambda + \frac{2}{3}G)\theta\delta_{ij} + \frac{\sigma_u}{\varepsilon_u}e_{ij} \quad (5)$$

where λ, G – coefficients Lame, σ_u, ε_u – respectively, intensity of tension and deformations connected by a ratio

$$\sigma_u = 2G(1-\omega)\varepsilon_u \quad (6)$$

In the last ratio function of plasticity of Ilyushin for material with linear hardening has an appearance

$\omega = \bar{\lambda}(1-\frac{\varepsilon_s}{\varepsilon})$, where $\bar{\lambda} = (1-\frac{E'}{E})$ - hardening coefficient.

Substituting in (5) values σ_u from (6), after some transformations we receive:

$$\sigma_{ij} = \lambda\theta\delta_{ij} + \frac{2}{3}G\theta\delta_{ij} + 2Ge_{ij} - 2Ge_{ij}\omega \quad (7)$$

Grouping the second and third members of the last equality and considering, as, we

$e_{ij} + \frac{1}{3}\theta\delta_{ij} = \varepsilon_{ij}$ and $\theta = \varepsilon_{kk}, \varepsilon_0 = 3\theta$, from which

$$\sigma_{ij} = \lambda\varepsilon_{\hat{e}\hat{e}}\delta_{ij} + 2G\varepsilon_{ij} - 2G(\varepsilon_{ij} - \delta_{ij}\varepsilon_0)\omega \quad (8)$$

Having entered designation:

$\bar{S}_{ij} = 2G(\varepsilon_{ij} - \delta_{ij}\varepsilon_0)$, a ratio (8) it is possible to write down in a look

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2G\varepsilon_{ij} - \bar{S}_{ij}\omega \quad (9)$$

Such form of record is convenient for realization of iterative process of a method of elastic decisions of Ilyushin A. A. Zones of plastic deformations are defined on the basis of Mises's criterion.

For the solution of a task its variation statement allowing to apply FEM and to receive the allowing system of the equations is considered. The solution of a task is carried out on the basis of a FEM in movements [8]. Accuracy of iterative process is defined by a ratio [9]: $\frac{\{\Delta u_i\}'\{\Delta u_i\}}{\{u_i\}'\{u_i\}} \le \varepsilon$, where $\varepsilon = 10^{-4}$.

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4 Algorithm of the decision and software

The computing algorithm of the solution of a task includes the following stages:
- creation of finite elements model;
- formations of system of the linear equations;
- decisions of system of the equations;
- definitions component of an elastic tension;
- solution of a nonlinear task;
- calculation of resultant parameters;
- visualization of results of calculation.

The above described stages of algorithm correspond to the settlement scheme of a FEM and pickings and storages of a big data file used in the course of calculation are necessary for automation of processing. Besides, such approach allows consolidating the solution of an objective to the stage-by-stage solution of independent simple tasks.

The ARPEK program complex consisting of a set of functionally connected program modules is developed for research of nonlinear deformation of fibrous elements of designs:

APKEM - automation of creation of finite elements discrete model,

NERPEK - constructions and decisions of the allowing system of the equations,

TASVIR - visualization results of calculation.

Besides, ways of interaction between program modules are worked. Modules can be packed easily among themselves, and also quickly be adjusted to specific conditions of a task that significantly reduces terms of development of a program complex. The modular structure of a complex allows to develop and improve separate modules, and to use them at various stages of the solution of a task. Such structure of a complex allows automating process of calculations, since a problem definition and finishing visualization of results of calculation.

For approbation of computing algorithm and program complex results of the solution of test tasks were verified with earlier known results [10], and also with the results received by means of a COSMOS/M package.

5 Computer modeling

5.1 Problem One

The task about bilateral stretching of a rectangular plate of single height, 0.5 cm wide and 0.05 cm thick is considered. In the middle of a plate two round hole with R=0.05 cm radiuses are vertically located. Mechanical parameters:

$E=2*10^5$ MPa, $\mu = 0.3$, $\bar{\lambda} = 0.5$, $\varepsilon_s=0.85*10^{-3}$.

The enclosed loading P_{zz} = 85 MPa. The task is considered in three-dimensional statement and is connected with research of influence of distance openings (h) on an elastic-plastic condition of a body

[11] between centers. By means of carrying out computing experiments the tension, depending on change of distance between holes is investigated at h = 0.5 cm, 0.35 cm and 0.2 cm (fig.1). The analysis of results of calculation shows that zones of the increased tension concentrate openings on each side, and lowered – over and under hole. At h = 0.5 cm development of a zone of the increased tension in the direction of plate corners is observed. At h = 0.2 cm – formation between openings of uniform area with the lowered tension. In process of reduction of distance between holes the increase in interference of hole is observed. It is reflected in the form of merge of areas to the increased tension in lateral parts of hole. In Table 1 the maximum values of intensity of tension σ_i^{up} and coefficients of concentration of tension $K=\sigma_{max}/\sigma_{nom}$ are presented, for an elastic task. From where it is possible to notice that with reduction of distance between holes also the values corresponding to them decrease.

Table 1. Maximum values of parameters

h [cm]	0.5	0.4	0.3	0.2
$\sigma_i^{up}/E*10^{-3}$	1.265	1.228	1.184	1.130
K	2.976	2.890	2.786	2.659

Further results of elastic-plastic calculation are given. In fig.1 the plate tension concerning intensity of deformations is reflected. On each side hole areas of plastic deformations are formed. With reduction of distance of h between holes the sizes of these areas and tension parameters corresponding to them also decrease (Tables 2, 3.).

Table 2. Maximum values component

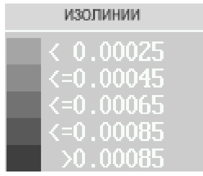
h [cm]	$\sigma_{xx}/E*10^{-3}$	$\sigma_{yy}/E*10^{-3}$	$\sigma_{zz}/E*10^{-3}$
0.5	0.9443	0.7240	2.208
0.4	0.7580	0.5248	1.893
0.35	0.7659	0.5244	1.841
0.3	0.7549	0.4997	1.758
0.2	0.6202	0.3805	1.421

Table 3. Maximum values component

h [cm]	$\tau_{zx}/E*10^{-3}$	$\sigma_i/E*10^{-3}$	ε_i*10^2
0.5	-0.5492	1.596	0.2061
0.4	-0.3726	1.292	0.1542
0.35	-0.2506	1.278	0.1527
0.3	-0.2263	1.218	0.1425
0.2	-0.3153	1.091	0.1234

Distribution research shows a component of tension, areas with the increased tension settle down in the lower lateral part of a hole. It is connected with interference of the hole located on one vertical.

Table 4. Compliance of tone a shading to values intensity of deformations



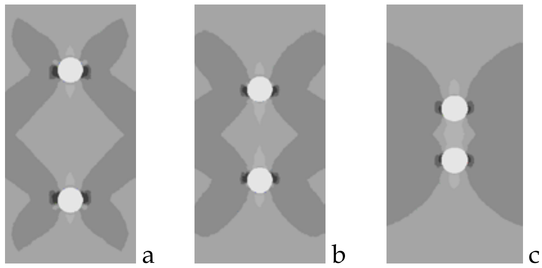


Fig. 1. Distributions of intensity of deformations

It should be noted that in this case there is a change of a tension of all hole in general. Influence of external loading is distributed on all perimeter of an hole. The tension picture round two vertically located holes is identical to a picture of a condition of a plate with one isolated hole.

As problems are solved in three-dimensional statement, in Table 2 also values components of normal tension σ_{yy} are presented. The plate height relation to its thickness makes 1:10. Relatives in value size σ_{yy} and values τ_{zx} in the vicinity of an hole specifies that in calculations of stretching of a plate with openings it is necessary to consider also this parameter.

5.2 Problem Two

With a research objective of influence of system of holes on a tension of elements of designs uniaxial stretching on an axis of OZ of an infinite rectangular strip of single width, with system of vertically located two holes is considered. Radiuses of holes of R = 0.05 cm, distance between holes across l = 0.5 cm. For research of influence and interference of distance (h) between vertically located holes, in elastic-plastic statement the problem is solved in two holes: h = 0.4 cm and 0.2 cm. Existence of system of holes conducts to formation of areas of comprehensive stretching of holes on each side across. So at h = 0.4 cm (fig.2) local zones of the increased tension in vicinities of lateral parts of holes are formed [12].

Areas with plastic deformation which configuration represents a form of "petal", are located in vicinities of the lower lateral parts of holes. Interference of holes down nominal as areas with the increased tension represents a picture similar to corresponding tension of system with the isolated hole. Interference of openings conducts to reduction of values of the maximum tension approximately for 10% across.

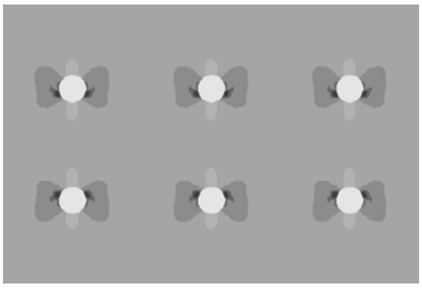


Fig. 2. Interference of holes (h = 0.4 cm)

Further results of calculation, when distance down between holes h = 0.2 cm, and distance across of l = 0.5 cm (fig.3 are investigated). Reduction of distance down leads to redistribution of a tension in vicinities of holes twice. Localization of the increased tension in vicinities of lateral parts of holes is also connected with existence of system of holes across, and change of a configuration of area happens owing to interference of vertical holes. It should be noted that in a case when distance down between holes in proportion to diameter of holes, zones with the increased tension extend across. This phenomenon can lead further to loss of durability of all of designs in general.

Thus, carrying out computing experiment allows on the basis of modeling of the solution of an elastic-plastic task to study influence and interference of vertically located openings on redistribution of the intense deformed state, and by a choice of a site of holes to keep strength characteristics of elements of designs.

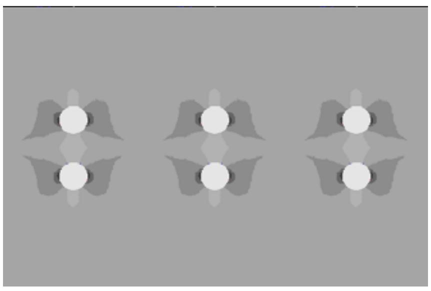


Fig. 3. Interference of holes (h = 0.2 cm)

5.3 Problem Third

The elastic-plastic condition of a plate with concentrators is considered, namely: with one and two vertically located round holes [13, 14]. The plate of single height, single axis stretches on an axis of OZ evenly distributed loading of P_{zz} = 85 MPa (width of a plate 0.5 cm and thickness – 0.1 cm). Holes have a circle form with identical radiuses of R=0.05 cm. Mechanical parameters of isotropic material:

$$E=2*10^5 \text{ MPa}, \mu = 0.3, \bar{\lambda} =0.8, \varepsilon_s=0.85*10^{-3}.$$

Problems are solved in three-dimensional statement by of FEM [15] together with method of elastic decisions of A. A. Ilyushin [16]. Influence and interferences of concentrators on the intense deformed state is investigated. For visualization of a form of not deformed and deformed finite-elements grid to values of movements the increase coefficient equal 400 is set.

At the initial stage of researches we will give the maximum values of intensity of tension and coefficient of concentration of tension of the above described task in elastic statement (Table 5). The analysis of results confirms emergence of effect of unloading at establishment of additional holes [17,18]. The successful arrangement of concentrators allows to reduce concentration of tension and to unload dangerous zones of a design. So, for example, establishment of two vertically located holes instead of one isolated hole allows reducing the maximum value of intensity of tension by 10%.

Further the solution of the above described task in elastic-plastic statement is considered.

Table 5. Values of parameters of a tension

Concentrator	$\sigma_i/E*10^3$	$K = \sigma_i / \sigma_{nom}$
Isolated hole	1.26	2.967
Two holes	1.13	2.690
Two holes +crack	1.09	2.565

In Tables 6, 7 the maximum values a component normal σ_{xx}, σ_{zz} and tangent τ_{xz} tension, intensity of tension and deformations for concentrators in the form of the isolated round hole, vertically located two holes, two holes with a rectilinear crack between them are given. The analysis of results will be carried out together with research of distribution of values a component of tension and deformations.

Table 6. Maximum values component of tension

Concentrator	$\sigma_{xx}/E*10^4$	$\sigma_{zz}/E*10^3$	$\tau_{xz}/E*10^4$
Isolated hole	9.451	1.634	-0.121
Two holes	7.099	1.565	-1.643
Two holes+crack	5.759	1.511	-1.839

Table 7. Maximum values component of tension

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Concentrator	$\sigma_i/E*10^3$	ε_i*10^3
Isolated hole	1.010	1.086
Two hole	1.048	1.153
Two holes +crack	1.064	1.203

Research of distribution of values a component of normal tension σ_{xx} (fig.4) specifies that in cases of two holes without and with a crack between them, holes work as the uniform concentrator.
The picture of distribution of fields corresponds to the phenomenon of unloading of areas between holes, and existence of a crack strengthens this effect (fig.4.b). It should be noted that the concentrators located in the center of a plate are in poorly intense zone (fig. 4.c).

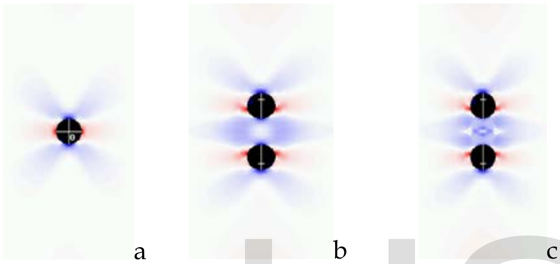

Fig. 4. Distribution of normal tension σ_{xx}

The same is observed and at research of distribution of values components of normal tension σ_{zz} (fig. 5a), and in a quantitative sense, rather isolated hole in the center, in case of two holes reduction makes 4.2% (fig. 5.b) and 7.5%-in the presence of a crack between holes (fig. 5.c). In quantitative sense values components of tension σ_{zz} increase on average by 18%, however this increase is generally connected with proximity of adjacent openings to areas where the stretching external loadings are enclosed.

Significantly results of research of distribution of values components of tangent tension τ_{xz}, given on fig.6 differ from above-stated. The picture has cardinal differences. If in the presence of the isolated hole areas in vicinities of lateral parts of a hole (fig.6.a) are subject to shift changes, in cases with two holes – the picture is turned on 90 ° (fig.6.b). Existence of a crack between holes reduces influence of shift loadings a small (fig.6.c).

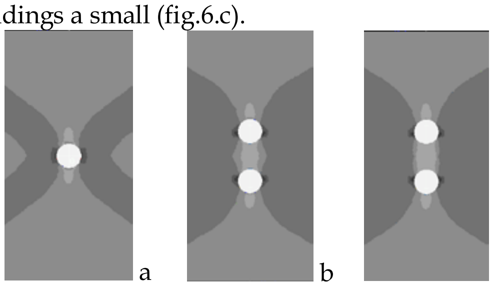

Fig. 5. Distribution of normal tension σ_{zz}

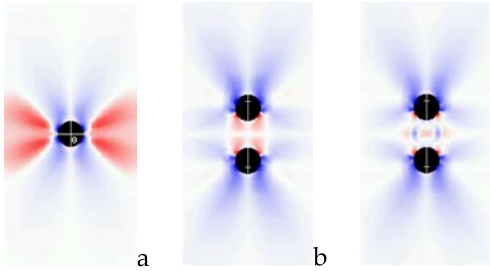

Fig. 6. Distribution of tangent tension τ_{xz}

For synthesis of the above-stated researches areas of distribution of intensity of deformations ε_i are given in fig. 7. Research of a zone of plastic deformations specifies that in the presence of the isolated hole the areas located in lateral parts of an hole (fig. 7.a) are subject to plastic deformations. The direction of the increased deformations makes a corner in 45° concerning the center of an hole. The areas located in the vicinity of the top and lower parts of holes are unloaded. In the presence of two holes with and without crack between them areas with the increased deformations increase, plastic zones concentrate and displaced to the center of a hole (fig. 7 b, c). The areas located between holes and from outer sides are unloaded. Existence of a crack expands areas with the increased deformations.

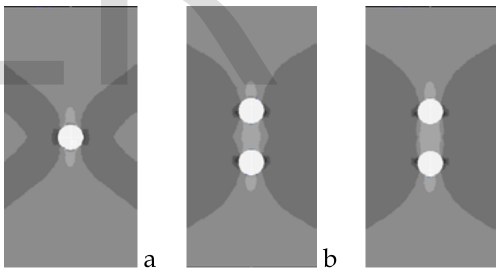

Fig. 7. Distribution of intensity deformations ε_i

Considering the unloading phenomenon at use of additional concentrators in elastic designs, it is possible to note that in the presence of two holes plastic deformations cause redistribution of a tension. Redistribution is connected with change of fields of distribution of tangent tension τ_{xz} and sharp increase in their values. Zones of plastic deformations are concentrated and displaced to the center of a plate. All this, in turn, leads to increase in intensity of tension. It is also necessary to note that values a component of normal tension σ_{xx} and σ_{zz}, as well as in case of an elastic task, decrease.

The received results correspond to physics of process and are checked on test tasks. For the solution of elastic-plastic tasks the method of elastic decisions of Ilyushin for the strengthened materials, the accuracy of iterative process $\varepsilon = 10^{-4} \div 10^{-5}$ is used. The number of iterations depends on concentration of plastic deformations and changes ranging from 4 to 8.

6 Conclusion

In the conclusion it should be noted that all above-stated results are received by means of carrying out computing experiment on the basis of a specialized complex of the ARPEK programs – automation of calculations of spatial elements of designs [19]. The analysis of results of the intense deformed state, allows defining an optimum location of additional openings, to reduce concentration of tension and to unload vicinities of concentrators.

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